erosion; and the occurrence of such forms beneath heavy limestones, 600 feet or more in thickness, clearly demonstrates the submergence of a previously eroded volcanic mass by over 600 feet, while the limestones were forming. Thus not only the recent history of the present barrier reef around Vanua Mbalavu, but also the Pleistocene history of the now dissected almost-atoll, of which Vanua Mbalavu is a remnant, testifies unqualifiedly in favor of Darwin's theory of coral reef and against all other theories. [Since the above was written Foye[3] gives independent evidence of the eastward tilting of Lakemba, which is on about the same meridian as Vanua Mbalavu.]

[1] J. S. Gardiner, *Cambridge, Eng., Proc. Phil. Soc.*, **9**, 1898, (417–503).
[2] A. Agassiz, *Cambridge, Mass., Bull. Mus. Comp. Zool., Harvard Coll.* **33**, 1899, (1–167).
[3] W. G. Foye, *Amer. J. Sci., New Haven*, **43**, 1917, (343–350).

STUDIES OF MAGNITUDE IN STAR CLUSTERS, VII. A METHOD FOR THE DETERMINATION OF THE RELATIVE DISTANCES OF GLOBULAR CLUSTERS

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More than 150 variables for which the light changes are rapid and periodic have been found among the thousand brightest stars in the globular cluster Messier 3. Eighty per cent of them were discovered twenty years ago by Professor Bailey at Harvard,[1] and the remainder three years ago by the writer at Mount Wilson.[2] The light variations of these stars are typical of a large class of variables—the short period Cepheids—some of which are found among the stars in the sky at large, though the far greater majority of those now on record are confined to a few of the globular clusters and to the Magellanic clouds. Wherever found they appear remarkably alike in range of variation, spectral type, color variation, length of period, nature of light changes, and even in the irregularities of the periods and the fluctuations of the light curves.

Recent work with the 60-inch reflector on the variables in Messier 3 is supplemental to the determination of light curves and periods by Bailey,[3] and is incorporated in the general study of magnitudes in clusters primarily for the intercomparison, on the basis of the Mount Wilson scale of magnitudes, of the brightness of variables in this and other globular systems. It is part of an investigation of the magnitudes and colors of all the brighter stars in Messier 3, and follows the methods previously employed.[4]

As much of Bailey's work preceded the establishment of the North Polar Sequence, his values of the magnitudes and ranges of variation may now be standardized. A sample of the revised data is given in table 1. The variables differ very little from each other in any respect, and particularly significant is the striking similarity of the median magnitudes. Bailey derived light curves and periods for 110 variables in this cluster, and 54 of them were chosen as fairly free from uncertainty. The median magnitude (defined as the mean of maximum and minimum) of these 54 stars is 15.49, on the Mount Wilson system, with a probable error of less than a hundredth of a magnitude. The average deviation from this mean for a single star is ± 0.07, and the largest deviation is less than two-tenths of a magnitude. If all 110 variables are used, the mean is 15.50 ± 0.006, and the average deviation is ± 0.08. The distribution of the residuals follows the law of error as closely as could be expected for a small number of values.

TABLE 1

STAR	PERIOD	BAILEY'S MAGNITUDE		MOUNT WILSON PHOTOGRAPHIC MAGNITUDE			MEDIAN MAGNITUDE
		Maximum	Minimum	Maximum	Minimum	Range	
	days						
9	0.542	14.60	16.76	14.82	16.15	1.33	15.48
10	0.570	14.75	16.70	14.92	16.11	1.19	15.52
27	0.580	15.00	16.68	15.06	16.10	1.04	15.58
34	0.559	15.00	16.70	15.06	16.11	1.05	15.58
38	0.561	14.80	16.83	14.95	16.19	1.24	15.57
40	0.552	14.70	16.83	14.88	16.19	1.31	15.54
49	0.548	14.76	16.76	14.93	16.15	1.22	15.54
63	0.570	14.80	16.62	14.95	16.06	1.11	15.50
80	0.539	14.74	16.70	14.91	16.11	1.20	15.51

Great extremes of absolute brightness are known to exist in Messier 3—nine or ten magnitudes, at least. More than 20,000 stars fainter than these variables have been photographed at Mount Wilson, but no other variables of this or other types have been found among them. The result is confirmed by an examination of long-exposure plates at Harvard. The periodic light variations are apparently confined to a narrow interval of brightness.

Since the deviations of the median magnitudes from their mean are far within the errors of observation, the conclusion is forced upon us that in Messier 3 short-period variation is associated with stars of a very definite intrinsic luminosity. The situation of the variables in a distant globular system necessitates the equality of absolute as well as of apparent magnitudes. Moreover, the work on color, so far as it has

gone, implies that the spectra are all likewise strictly comparable. We conclude, therefore, that in surface brightness and volume, and probably in mass, density, and other physical properties, these 110 stars are almost identical.

Whether the isolated variables of this so-called cluster type also are exactly alike in luminosity, we have at present no means of knowing, other than analogy, because their distances are unknown and their apparent magnitudes differ greatly. To investigate further the possible generality of a law of constant median magnitude, some special studies of the variables in other clusters have been made. The extended discussion will appear in the *Astrophysical Journal.* The results are summarized in the following numbered paragraphs.

1. In Messier 5 Bailey has recently determined the periods and light curves of about 70 variables.[5] Excluding those that are nearer the center than 1′.2, the results for which must be uncertain because of the crowding of images, and omitting also three for which the periods exceed a day, the median magnitude of the remaining 61, referred to Mount Wilson standards, is 15.26±0.01, the average deviation for a single star being ±0.075. Considering only the 30 light curves selected by Bailey as well-determined, the mean median magnitude is 15.25, with an average deviation of ±0.08.

2. In Messier 15 are 51 known variables,[1] but the light curves have not yet been determined. On three plates the extreme magnitude range of the variables has been measured and found to be:

Maximum	14.98	Extremes of range	1.22
Minimum	16.20	Median magnitude	15.59

Two peculiarly bright stars, suspected of variability, are excluded. The range of variation is in very good agreement with the mean ranges for Messier 3 and 5, and, although the results are not final, the median magnitude is probably correct within a tenth of a magnitude.

3. In the southern cluster ω Centauri three subclasses of cluster variables are recognized. Treating each separately we have, for the stars whose classification is certain, the results of table 2, which are

TABLE 2

SUB-CLASS	NUMBER VARIABLES	MEAN PERIOD	MAXIMUM MAGNITUDE	RANGE OF VARIATION	MEDIAN MAGNITUDE	AVERAGE DEVIATION
		days				
a	33	0.586	12.99	1.11	13.55	±0.09
b	15	0.752	13.11	0.87	13.55	±0.10
c	28	0.395	13.33	0.56	13.61	±0.09
All	76				13.57	±0.10

taken from Bailey's tabulation with only slight modifications and additions.[6] Subclass *a*, as the range of variation suggests, is the type that prevails in Messier 3 and 5. Although the stars of the three groups differ from each other in maximum and in range, as well as in period, the median magnitude is the same. The distribution of deviations again follows closely the probability curve. As before the short period variables are restricted to a small interval of brightness which can be represented with high accuracy by a mean median magnitude. The whole interval observed in ω Centauri is six or seven magnitudes, but no fainter variables are found, and the two or three brighter ones are long-period Cepheids. The magnitudes used for this cluster are not referred to the Mount Wilson system, and, therefore, are not strictly comparable with those of the other clusters. They are probably about right, although the uncertainty may be as much as half a magnitude.

4. Ten stars in Messier 2 have been suspected of variation.[1] Eight have been verified on Mount Wilson plates, but as no comparisons with the Pole have been made, only provisional results are available. Measures on three plates give the extreme variation as one and a third magnitudes, agreeing with all that precedes in showing that the short period variables are confined to a definite limit of magnitude.

These results for the variables in five clusters have an obvious application in the determination of relative parallaxes. We need only the hypothesis, apparently reasonable in the light of the foregoing discussion, that the absolute median magnitudes, which appear so constant in each cluster, are actually identical in all systems. The observed differences in the mean values then become sensitive criteria of distance, and the relative parallaxes of these remote systems can be known with an accuracy which will depend only on the precision with which the photographic magnitudes can be determined.

For instance, we find above that the median magnitude for Messier, 3 is 15.49 ± 0.01, and the corresponding value for Messier 5, 15.25 ± 0.01. Possibly there are small systematic errors due to choice of variables, to errors in the maxima, or to remaining errors of zero point, which are not eliminated in taking the difference. But certainly we know the difference in the two median magnitudes within a tenth of a magnitude, and consequently the difference in distance of these remote and nearly equidistant clusters within 40% of its value; while the difference in distance can be known within 7% of its value, if either cluster is compared with ω Centauri. And once we find the absolute median magnitude of such variables—by no means a hopeless task—the actual distances of all clusters with typical variables can be determined within

5%, an accuracy as yet quite unattainable by direct measurement for any stellar object except the nearest stars.

The derivation of a probable value of the absolute luminosity of cluster-type variables will be given in the extended paper. Provisionally we observe that the absolute median magnitude is probably within the limits -0.5 and $+1.5$ (unit of distance, $\pi = 0''.01$), and on that basis the absolute parallax of four of the above clusters is as follows:

Messier 3 .... π between 0".00006 and 0".00016
Messier 5 .... π between 0.00007 and 0.00018
Messier 15 .... π between 0.00006 and 0.00015
ω Centauri .... π between 0.00015 and 0.00038

Only a small number of the globular clusters are known to contain short-period variables. The relative distances of the others can be estimated, however, on the basis of a relation, found in the clusters that contain variables, of the median magnitude to the average magnitude of the brightest stars. To derive this relation the photographic magnitudes of all the brighter stars in each cluster were determined, excluding stars more distant from the center than 10′ as possibly not members, and also these within 2′ as too liable to uncertainty of measurement. Of those remaining, the first five in order of brightness were discarded as superposed stars, or as non-typical in luminosity. The mean magnitude of the next 25 stars, a homogeneous group of highly luminous objects, was formed. Thus were obtained the results of table 3, which seem as remarkable and as significant for stellar theories as the phenomenon of constant median magnitude.

TABLE 3

CLUSTER	MEDIAN MAGNITUDE	AVERAGE DEVIATION	NUMBER OF VARIABLES	MEAN MAGNITUDE BRIGHTEST 25 STARS	AVERAGE DEVIATION	MEDIAN MINUS BRIGHTEST	WEIGHT
Messier 3	15.50	±0.08	110	14.14	±0.16	1.35	4
Messier 5	15.26	±0.075	61	13.92	±0.15	1.34	2
Messier 15	15.59		49	14.28	±0.18	1.31	2
Messier 2			8		±0.22	1.40	1
Weighted mean difference						1.35	

In view of these results it is reasonable to believe that, if short-period variables did exist in one of the many globular clusters in which they have not been found, their median magnitudes would average about 1.35 fainter than the average magnitude of the 25 selected brightest

stars. In other words, it is proposed to estimate the distances of clusters lacking typical variables on the basis of their bright stars.

Finally it may be noted that our assumption of the identical luminosity of these variables in the various clusters tacitly implies that the corresponding variables in the galactic system are all of the same absolute magnitude. That leads immediately, and without further hypothesis, to the derivation of highly precise relative parallaxes of the 40 scattered stars of this type, and as some of these variables are extremely faint, the results bear directly on the extent of the general stellar system.

Summary.—(1) The median magnitude of the short-period variables apparently has a rigorously constant value in each globular cluster. (2) These stars also possess essentially identical spectra and color variations. (3) Such phenomena, implying remarkably similar physical conditions for all cluster-type variables, must play an important part in theories of Cepheid variation. (4) The present use of the photographic magnitude observations, however, is to derive accurate relative distances of globular clusters on the basis of the differences, from system to system, of the mean median magnitude. (5) The method extends naturally to all isolated cluster-type variables in the galactic system. (6) By means of a relation between the median magnitude and the magnitudes of the brightest stars in a cluster, the relative parallaxes of the many clusters lacking variables may be estimated with considerable assurance when the magnitudes have been measured. (7) The determination of the absolute luminosity of some of these variables will give immediately absolute individual distances of the clusters and of the forty isolated cluster-type variables of known magnitude and period. (8) A preliminary value of this absolute median magnitude indicates that with one or two exceptions no globular cluster is nearer than thirty thousand light-years ($\pi = 0''.00012$).

[1] Bailey, S. I., *Ann. Obs. Harvard Coll., Cambridge*, **38**, 1902, (1–252), page 2.

[2] Shapley, H., *Mt. Wilson Contrib.*, No. 91; *Astroph. J., Chicago*, **40**, 1914, (443–447).

[3] Bailey, S. I., *Ann. Obs. Harvard Coll., Cambridge*, **78**, Part I, 1913, (1–98).

[4] Shapley, H., *Mt. Wilson Contrib.*, No. 115, 1915, (1–92), pages 8 to 22.

[5] Bailey, S. I., *Ann. Obs. Harvard Coll., Cambridge*, **78**, Part II, 1917, (103–192).

[6] *Ibid.*, **38**, 1902, (1–253).